Advanced Composite Materials
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tacm20

Advanced probabilistic design and reliability-based design optimization for composite sandwich structure

Seokje Lee[a], In-Gul Kim[a], Wooje Cho[a] & Changwon Shul[b]

[a] Department of Aerospace Engineering, Chungnam National University, 99 Daehak-ro, Yuseong-gu, Daejeon 305-764, Republic of Korea

[b] Agency for Defense Development, Yuseong-gu, P.O. Box 35-7, Daejeon 305-600, Republic of Korea

Published online: 07 Dec 2013.

To cite this article: Seokje Lee, In-Gul Kim, Wooje Cho & Changwon Shul (2014) Advanced probabilistic design and reliability-based design optimization for composite sandwich structure, Advanced Composite Materials, 23:1, 3-16, DOI: 10.1080/09243046.2013.862381

To link to this article: http://dx.doi.org/10.1080/09243046.2013.862381

Advanced Composite Materials, 2014
Vol. 23, No. 1, 3–16, http://dx.doi.org/10.1080/09243046.2013.862381

Advanced probabilistic design and reliability-based design optimization for composite sandwich structure

Seokje Lee[a], In-Gul Kim[a]*, Wooje Cho[a] and Changwon Shul[b]

[a]*Department of Aerospace Engineering, Chungnam National University, 99 Daehak-ro, Yuseong-gu, Daejeon 305-764, Republic of Korea;* [b]*Agency for Defense Development, Yuseong-gu, P.O. Box 35-7, Daejeon 305-600, Republic of Korea*

(*Received 27 December 2012; accepted 1 April 2013*)

A composite sandwich structure can improve flexural rigidity and can decrease the weight of a composite laminated plate by 30%. However, it implies significant uncertainty within the construction process of its material properties when compared to other regular metals. Therefore, it is necessary to consider a probabilistic design method based on reliability. This paper calculated the probabilistic margin of safety of a simplified composite sandwich fuselage in order to examine that the classic design method utilizing the safety factor does not ensure the safety of the structure. Reliability-based design optimization (RBDO) was conducted for efficient calculation by using proposed RBDO by moving probability density function (RBDO-MPDF) method in this paper. Crude Monte-Carlo Simulation was used to calculate the probability density function. The results of this paper will be applicable to the improved design methods that ensure the structural reliability and maximized efficiency within the RBDO process.

Keywords: composite sandwich; uncertainty; reliability analysis; probabilistic margin of safety; RBDO by moving PDF

1. Introduction

The use of composite materials is ever increasing with its application in commercial airplanes and even in military and space flight vehicle structures. In particular, the application of the composite sandwich structure within aerospace structures can dramatically increase the mechanical property as well as ameliorate the performance in comparison to heat insulation and fatigue life.[1]

However, not only are the high costs and the difficulty of the manufacturing process problems facing the material, but also there is inherent uncertainty in the process of construction (the impregnation rate of matrix, the presence or absence of incontinuous space, the thickness of each ply, etc.) and hence the application of the material is not easy. Till today, much of the practices are conducted through gathering design allowables based on experiments and are dependent on this conservative design method. This entails significant economic and material consumption. Therefore, there is a need to introduce the probabilistic analysis method in order to develop a sandwich structure that is effective, reliable, and satisfies the various design requirements.[2,3]

A probabilistic analysis method that is based on reliability provides the required information for optimum design. Also, it brings rationality to the consideration of

*Corresponding author. Email: igkim@cnu.ac.kr

uncertainty in design, and can provide an effective design when applied to a professional experience and technique obtained from a specific system.[4] This method applies the distribution properties that are centered on the expected value of an external load or material properties quantitatively to calculate the probability of failure and ensure reliability. As shown in Figure 1, the classical method that uses the A/B-basis strength value and the safety factor results in a structure without failure. Yet, when considering the uncertainty factor, the stress due to the external load overrides the material strength and this produces an area that is failed.

In this paper, the probabilistic margin of safety (PMS) based on the reliability analysis was calculated and the RBDO by moving probability density function (RBDO-MPDF) for effective reliability-based design optimization (RBDO) was proposed ultimately. In order to verify this proposed method, a cantilever and a composite sandwich fuselage were used as examples. First, the distribution properties of the random variable are applied to the obtained results using the deterministic optimization (DO) in order to calculate the probability of failure and present the limitations of the classical design methods. Then the probability density function (PDF) was moved to a condition in which it will be under the probability where almost no failure occurs to calculate the PMS. From this, a revised design constraint was proposed to establish the reliability of the structure. Furthermore, in order to achieve an effective RBDO method, the initial value was determined as the average of the PDFs. This step was necessary to decrease repetitive procedures and to contemplate the effectiveness of the RBDO-MPDF methodology. In this process, the random variable was selected as the material stiffness, strength, and loading conditions.

2. Theoretical interpretation

2.1. *PMS*

Generally in the design of an aircraft structure, 1.5 of the safety factor is applied and the A/B-basis strength value is used to calculate the margin of safety (MS) through Equation (1).[5,6] Here, R_u is the ultimate strength of the material and S_u is the stress by ultimate load.

$$\mathrm{MS} = \frac{R_u}{S_u} - 1 \geqslant 0 \tag{1}$$

This method is a way of compensating for the uncertainties of the variables based on experiential evidence. It is a conservative method and there is no concrete evidence that it will ensure reliability.

Thus, this paper applied the uncertainty as in Figure 2 to obtain the PDF of the structural response (yield strength R_y, maximum stress S_I) and then move this by 3σ to propose a PMS ($\bar{R}_y/\bar{S}_I' - 1$) that almost does not cause any probability of failure. This

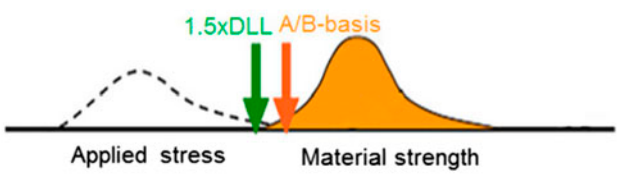

Figure 1. Conceptual diagram of deterministic and probabilistic concept.

proposed method devises a relatively simple concept and simple calculation process to compensate for the results of the conservation design method.

2.2. RBDO-MPDF

Equation (2) shows the general definition of the RBDO.[7] Here, d is the design variable vector, X is the random variable, $G_i(X)$ is the limit state equation, and β_{ti} is the desired probability of failure.

$$\text{Minimize cost}(d)$$

$$\text{Subject to } P(G_i(X) \leqslant 0) - \Phi(-\beta_{ti}) \leqslant 0, i = 1, \ldots, np \quad (2)$$

$$d^L \leqslant d \leqslant d^U$$

The major problem in this method is that too much computational cost is required to calculate the probabilistic constraint within the repetitive optimal process. So, generally, a first-order reliability method such as the advanced first-order second moment (AFOSM) is used [8]. However, this is a theory that likens the limit state equation to a linear function and therefore implies the error in the actual value. Hence, this paper deployed the Crude Monte-Carlo Simulation (CMCS) that can accurately calculate the probabilistic constraint, and in order to increase the efficiency, proposed the RBDO-MPDF with the average value of the moved PDF as the starting point. This method satisfied the probability of failure in moving the PDF (from light line to dark line in Figure 3), and the process of finding the optimal solution can be shortened by defining the structural value as the starting point. Figure 3 shows the conceptual diagram of the RBDO-MPDF and Figure 4 is the corresponding flow chart.

3. Numerical examples and results

3.1. Metal cantilever beam

This example is of a metal cantilever which is often used in reliability analysis and optimization problems. Figure 5 shows the configuration of a metal cantilever. The related variable's properties are appeared in Table 1.[9,10] Here, the coefficient of variable (COV) implies the ratio of the standard deviation.

In this example, the respective response function of stress and displacement as shown in Equations (3) and (4) can be defined as formulas.

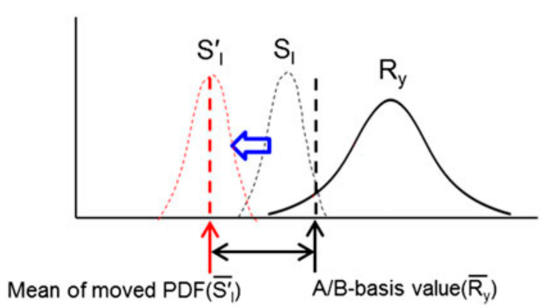

Figure 2. Conceptual diagram of PMS calculation.

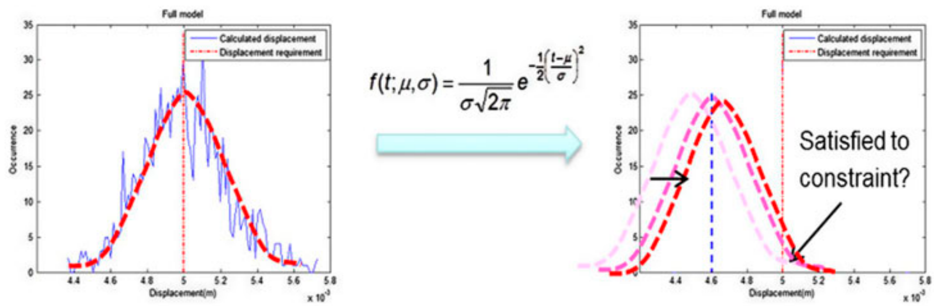

Figure 3. Conceptual diagram of RBDO-MPDF method.

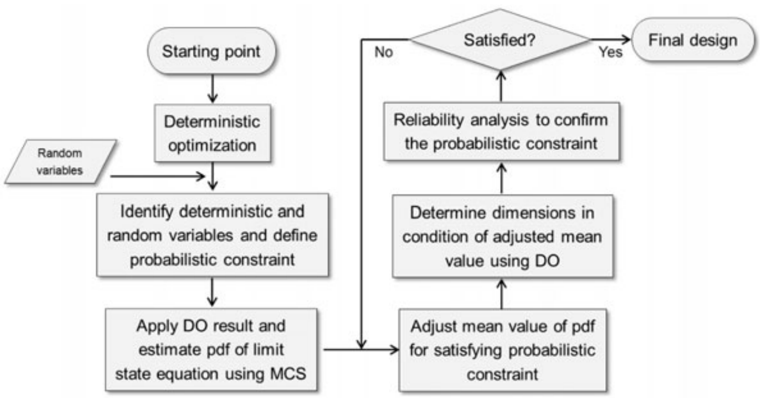

Figure 4. Calculation flowchart of RBDO-MPDF method.

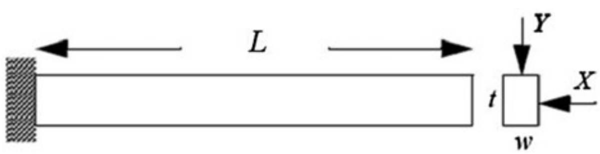

Figure 5. Configuration and symbols of metal cantilever example.

$$g_S = R - \frac{15,240}{wt}\left(\frac{Y}{t} + \frac{X}{w}\right) \tag{3}$$

$$g_D = D - \frac{4L^3}{Ewt}\sqrt{\frac{Y^2}{t^4} + \frac{X^2}{w^4}} \tag{4}$$

The design objective is to minimize the cross-section area within the probabilistic constraint. The case of applying the level 3.0 of reliability index (β) can be formulized as Equation (5). In the case of DO, the deterministic strength or displacement value is replaced as the constraint instead of the probabilistic constraint.

Table 1. Properties of variables (cantilever).

Variable	Symbol	Mean	COV	Distribution
X-direction force	X	2225 N	0.2	Normal
Y-direction force	Y	4450 N	0.1	
Yield strength	R_y	276 MPa	0.05	
Ultimate strength	R_u	400 MPa	0.05	
Young's modulus	E	200 GPa	0.05	
Length	L	2540 mm	–	–
Requirement for displacement	D	57.2 mm	–	–

$$\text{Minimize } w \times t$$
$$\text{Subject to } \beta \geqslant 3.0 \text{ or } P_f \leqslant 0.13\% \tag{5}$$
$$25.4\,\text{mm} \leqslant w, t \leqslant 101.6\,\text{mm}$$

The response function of the stress was used to calculate the PMS. The reason for this was to conduct the MS-based design using the safety factor and A/B-basis strength value as well as to compare with the probabilistic design method. First, the safety factor (1.5) was applied to each direction of the original load (design limit load) and DO was conducted using the A-basis value of the ultimate strength of the material as the deterministic constraint. When applying the random variable's uncertainty to the optimized cantilever's width (w) and thickness (t) as well as exerting the original load, the spread of the maximum stress will have a distribution such as Figure 6. Conclusively, the safety factor and A-basis value were used to overcome the uncertainty effects of the load and material property, but there still exists a range in which the maximum stress exceeds the yield strength and the cantilever structure's probability of failure was calculated as 8.75%. This is a result of conducting the CMCS 100,000 times and the confidence level is 95% with an error of ± 0.18%. Figure 7 shows the change in the probability of failure as the simulation is being processed. The

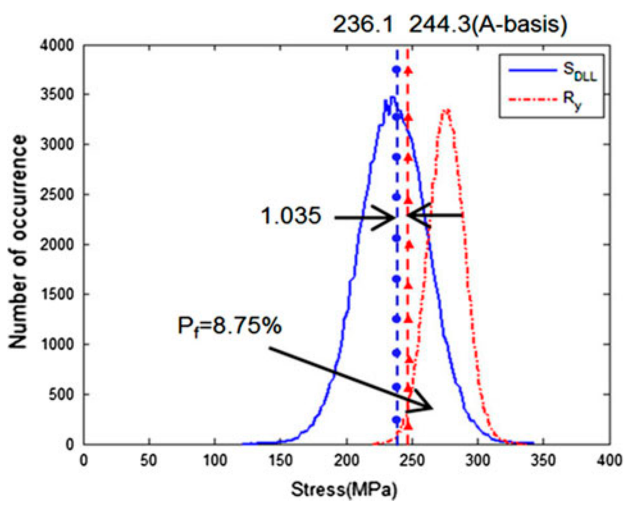

Figure 6. Distribution of MS-based design (cantilever, stress).

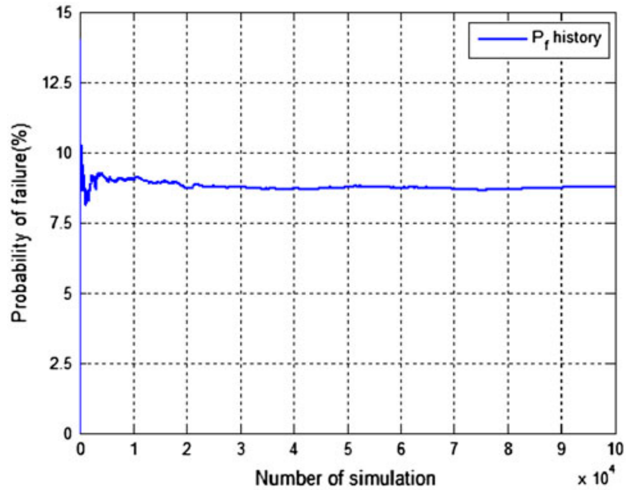

Figure 7. History of probability of failure (cantilever, stress).

probability of failure was well converged and that the number of simulation that was adequate was confirmed.

Therefore, when considering the uncertainty of the variable, the cantilever's cross-section dimensions need to be enlarged. Figure 8 shows the PMS that was obtained through the use of the ratio of the A-basis value of yield strength and the average of the moved stress distribution. The ratio of the two values is 1.56, so the PMS of the presented example is 0.56. That is, the maximum stress must be below 156.9 MPa for achieving a safe structure without failure. Table 2 compares the optimized result obtained through the classical method based on the MS and the optimized result based on the PMS with the result of reference [9] example.

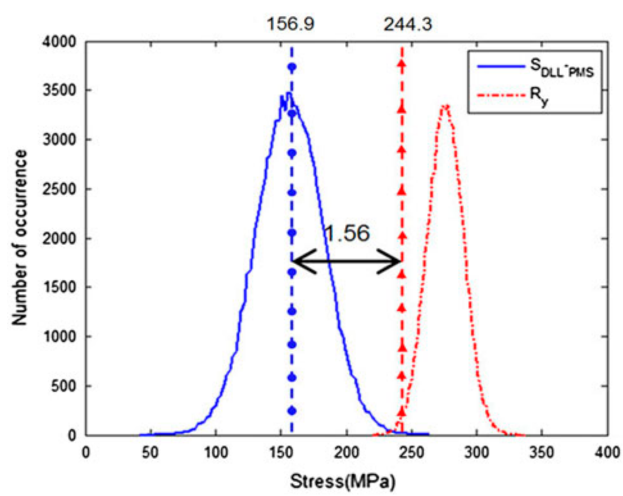

Figure 8. Distribution of PMS-based design (cantilever, stress).

Table 2. Comparison with MS- and PMS-based optimization (cantilever).

	w (mm)	t (mm)
MS-based DO	54.0	101.6
PMS-based DO	71.6	101.6
Reference	60.0	84.5

In this paper, each response function is used for the optimization, respectively. The result in the reference is subjected to two functions simultaneously. So it is difficult to compare the result directly. But in the previous study, the propriety of optimum algorithm of in-house code was confirmed.

The response function of the displacement was used in order to verify the RBDO-MPDF method. As in the case of the example that used stress, the safety factor was applied to each direction of the original load (design limit load) and DO was conducted using the displacement (*D*) as the deterministic constraint. The uncertainty of the random variables were applied to the width and thickness of the optimized cantilever, and upon applying the original load the maximum displacement has a distribution as shown in Figure 9. In this case as well, the CMCS was conducted 100,000 times and the probability of failure was calculated to be 0.015%. This example has a 95% confidence level and the error is ± 0.0077%. Figure 10 shows the changes in probability of failure throughout the duration of the simulation.

In order to conduct the RBDO-MPDF, the PDF of the distribution was first calculated. Then, the PDF was moved to satisfy the probabilistic constraint and estimate the average. This average value was then used to determine the starting point of the cross-section width and thickness, which ultimately was used to achieve optimization. From this starting point, the accurate probability of failure was calculated by using CMCS, and when the obtained result did not satisfy the constraint, then the design point was moved and the probability of failure was re-evaluated. This process was repeated until the values fall within the permissible range of error. As a result, it

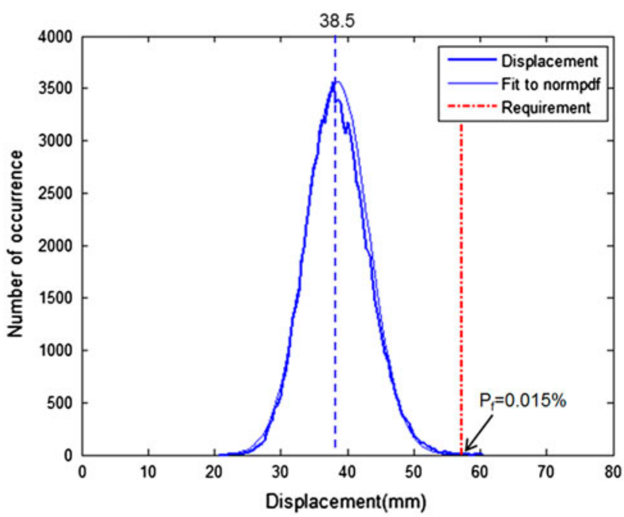

Figure 9. Distribution of MS-based design (cantilever, displacement).

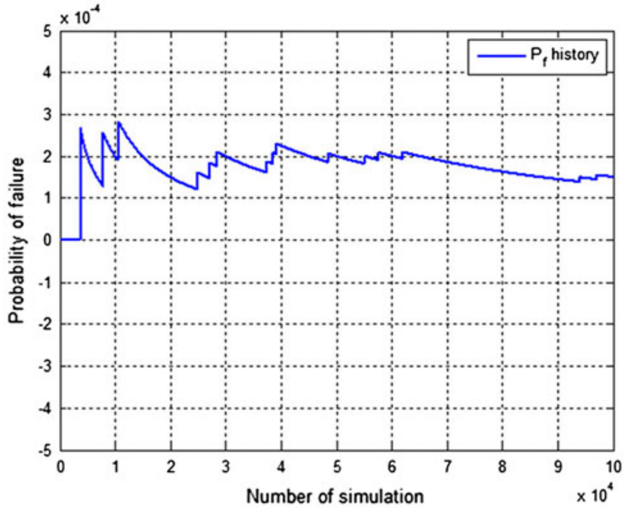

Figure 10. History of probability of failure (cantilever, displacement).

was observed that the PDF was moved to the right in order to achieve a 0.13% probability of failure as shown in Figure 11. The cross-section values that satisfy the aforementioned probability of failure are shown in third row of Table 3. Unlike the response function of stress, the classical method based on the MS leads to conservative results, whereas applying RBDO reduces the weight due to the reduction in the cross-section dimension. Also, even though the CMCS was used to obtain the probability of failure, the computation time for the calculations was shortened. The number of repetitions also came out to be four times and the efficiency of the RBDO-MPDF method can be amplified when applied to large structures that entail large random variables. The difference of optimized results with the general RBDO method can be come from the difference of used reliability analysis method for evaluating the probabilistic constraints. In other words, the RBDO-MPDF method uses the CMCS and the general RBDO method uses the AFOSM. Also, it can be seen that the multiplication of w and t of the objective function does not show much difference (Table 3).

3.2. Composite sandwich fuselage

This example involved the composite sandwich fuselage.[11] The configuration of a sandwich skin and a ring frame is shown in Figure 12. The sandwich structure is composed of face sheets that are laminated by T300 Carbon/Epoxy prepreg and aluminium honeycomb core. In addition, there is an aluminium ring frame bonded

Table 3. Comparison with deterministic and probabilistic optimization (cantilever).

	w (mm)	t (mm)	P_f (%)	Time (sec)
MS-based DO	66.1	93.5	–	1
General RBDO	68.6	86.6	0.13	18
RBDO-MPDF	65.1	92.1	0.13	4

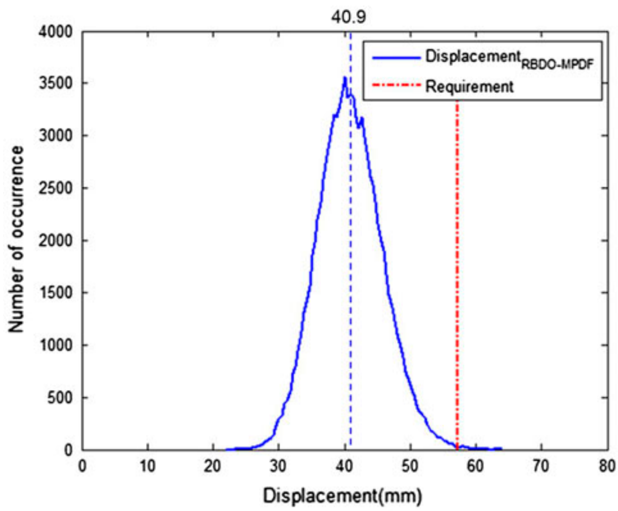

Figure 11. Distribution of RBDO-MPDF (cantilever, displacement).

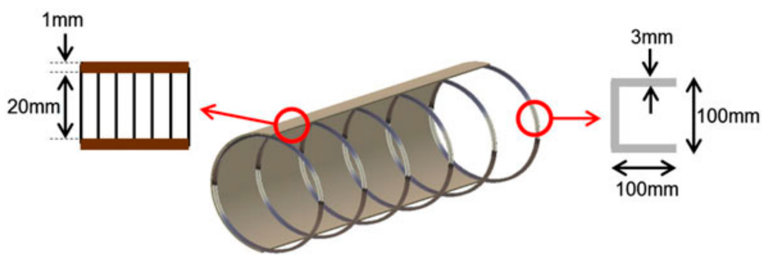

Figure 12. Configuration of composite sandwich fuselage.

jointed on to the lower face sheet. Figure 13 shows the finite element model in which the loads are imposed to the RBE2 element and all degree of freedoms on the other side are fixed. The stacking sequence of the face sheet was assumed to be $[0/+45/-45/90]_S$. The properties of random variables are shown in Table 4.[11,12]

The objective of optimization is to minimize the weight of the fuselage within the defined probabilistic constraint. The Tsai–Wu criterion was used and the Equation (6) was used to optimize the structure when the reliability index level 3.0 is applied. In the case of the DO, the constraint is replaced by the failure index 1 instead of the probabilistic constraint.

$$\text{Minimize weight}$$

$$\text{Subject to } \beta \geqslant 3.0 \text{ or } P_f \leqslant 0.13\% \tag{6}$$

$$0.125\,\text{mm} \leqslant \text{thickness of each ply} \leqslant 1\,\text{mm}$$

In order to calculate the PMS, the safety factor (1.5) was applied to each of the original load and optimization was achieved with the constraint that the failure index cannot exceed 1. In each of the upper and lower face sheets, plies with the same angle were

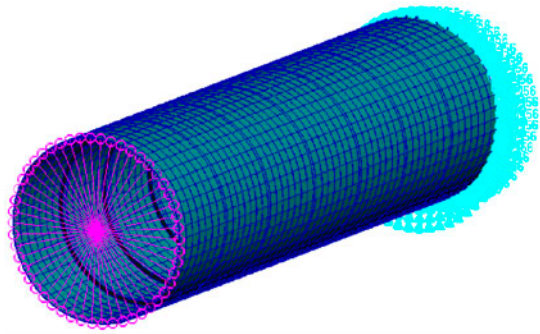

Figure 13. Finite element model of composite sandwich fuselage.

Table 4. Properties of variables (sandwich fuselage structure).

	Mean	COV	Distribution
E_1	133.92 GPa	0.046	Normal
E_2	8.84 GPa	0.040	
v_{12}	0.336	0.060	
G_{12}	4.45GPa	0.031	
X_T	1,787 MPa	0.14	
X_C	1,185 MPa	0.14	
Y_T	58.36 MPa	0.11	
Y_C	249.96 MPa	0.07	
S	106.93 MPa	0.02	
M_x (moment)	2.6×10^6 N·m	0.10	
M_y (moment)	9.5×10^6 N·m	0.10	
T (torsion)	9.0×10^5 N·m	0.10	
P (pressure)	0.13 MPa	0.10	

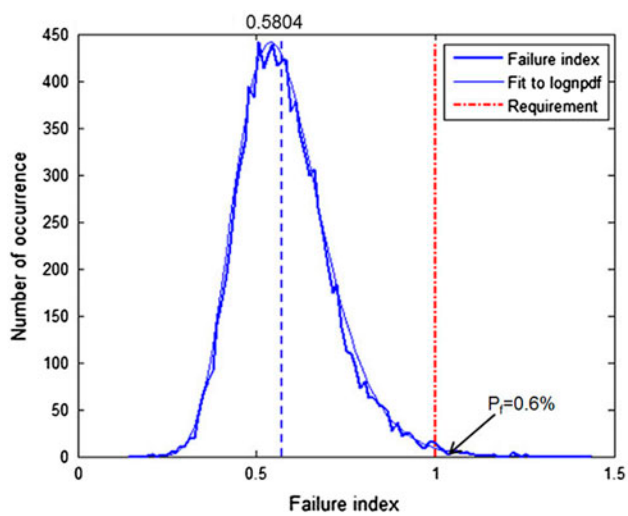

Figure 14. Distribution of MS-based design (sandwich fuselage structure).

treated as a single variable. When applying the uncertainty of the variables to the optimized result as well as the original load, the maximum failure index distribution is obtained. As a result, the region where the failure index exceeds 1 becomes the probability of failure, and as shown in Figure 14, the probability of failure was calculated as 0.6%. Ten thousand runs of CMCS were conducted with 95% reliability and ± 0.15% error. Also, as seen in Figure 15, it was observed that the failure probability converged as the simulation progressed. Unlike the example of the cantilever, a closed-form response function is unknown. So instead, every time the simulation by using the finite element analysis was conducted to compensate for the issue. Desktop PC (i7 CPU 920 and 6 GB RAM) was used in this study and computation time was 28 h.

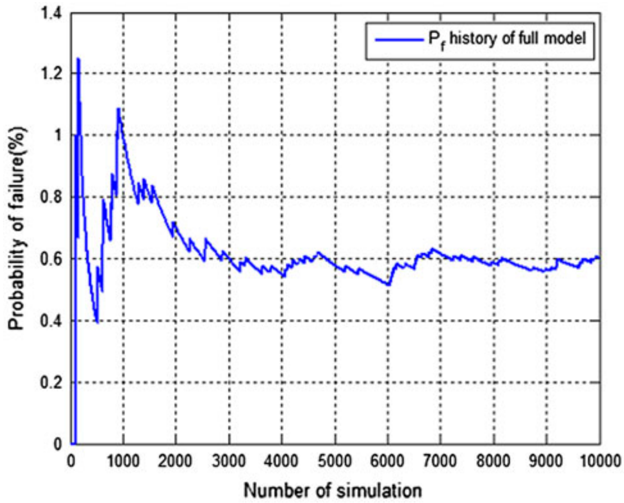

Figure 15. History of probability of failure (sandwich fuselage structure).

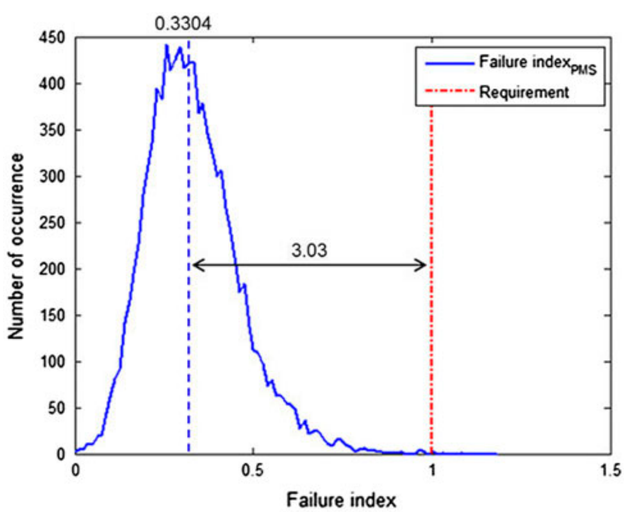

Figure 16. Distribution of PMS-based design (sandwich fuselage structure).

Table 5. Comparison with MS- and PMS-based optimization (sandwich fuselage structure).

	0°	±45°	90°
Upper facesheet (mm)			
MS-based DO	0.287 (0.375)	0.125	0.204 (0.250)
PMS-based DO	0.376 (0.500)	0.125	0.296 (0.375)
Lower facesheet (mm)			
MS-based DO	0.244 (0.250)	0.125	0.136 (0.250)
PMS-based DO	0.364 (0.375)	0.125	0.260 (0.375)
Weight (kg)			
MS-based DO		1015 (1116)	
PMS-based DO		1184 (1316)	

Note: *( ): Adjusted value in terms of single prepreg thickness (0.125 mm) in real practices.

Therefore, when considering the uncertainty, each of the thickness of ply within a face sheet has to be thickened. Figure 16 shows the ratio between the failure index 1 and the average of the moved failure index distribution. The ratio of the two values is 3.03, so the PMS of this example is 2.03. That is, the maximum failure index must be below 0.3304 to satisfy no failure and safe structure. Table 5 compares the optimized result using the classical method based on the MS and that obtained through the PMS-based method. In this optimization problem, all variables are handled as continuous design parameters. Therefore, adjusted values in terms of single prepreg thickness are added in round brackets to help you understand in real practices.

To confirm whether the sandwich failure occurs or not, analytically predicted stress was calculated.[13] The other typical failure stress such as the intra-cell dimpling and face wrinkling are 30.38 and 59.83 GPa. Maximum mean stress of face sheet is about 120 MPa when the thickness of MS-based DO result of Table 5 is applied. Therefore, there are enough margins for these failure stresses.

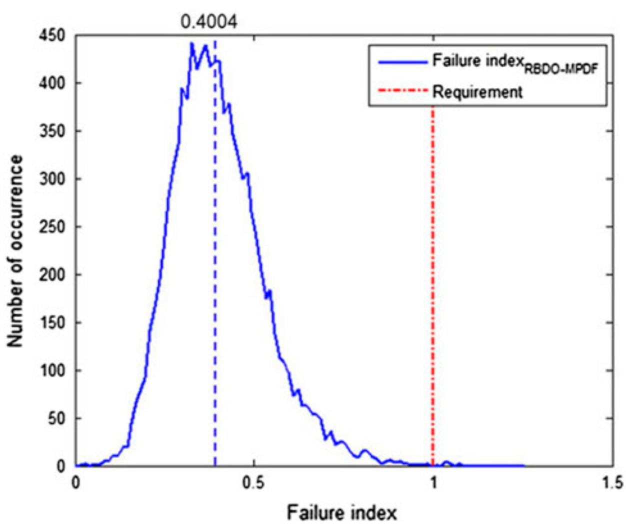

Figure 17. Distribution of RBDO-MPDF (sandwich fuselage structure).

Table 6. Comparison with deterministic and probabilistic optimization (sandwich fuselage structrure).

	0°	±45°	90°
Upper face sheet (mm)			
MS-based DO	0.287 (0.375)	0.125	0.204 (0.250)
RBDO-MPDF	0.336 (0.375)	0.125	0.244 (0.250)
Lower face sheet (mm)			
MS-based DO	0.244 (0.250)	0.125	0.136 (0.250)
RBDO-MPDF	0.315 (0.375)	0.125	0.228 (0.250)
Weight (kg)			
MS-based DO		1015 (1116)	
RBDO-MPDF		1115 (1166)	

Note: *( ): Adjusted value in terms of single prepreg thickness (0.125 mm) in real practices.

The process of PBDO-MPDF is as introduced previously. As a result, in order to achieve a probability of failure 0.13%, it was confirmed that the PDF was moved to the left as shown in Figure 17. The thickness required to satisfy the average value of 0.4004 of the moved PDF is summarized in Table 6. The process was repeated three times. The failure index was larger than the average calculated by the PMS, so the optimized thickness was relatively thinner. Finally, the ply with 0° angle played the most significant role in supporting the load based on the optimum result.

4. Conclusions

Probabilistic design method based on reliability analysis is perceived as the more effective and improved methodology. The proposed methods in this paper were applied to the optimal design of a metal cantilever beam and composite sandwich fuselage structure.

The paper first proposed PMS, which can guarantee structural safety even when applying the uncertainty that was calculated to improve the DO results. Also, for the effective RBDO of a large-scale and complex structures, the RBDO-MPDF method was proposed and its potential applicability was evaluated. As a result, the proposed method is effective and works well in the RBDO-MPDF for two numerical examples.

In the future, there is a need to evaluate the efficiency and accuracy of the system with the application of the approximate model such as response surface method. The results of this paper can be applied to research works in computationally effective optimization systems as well as multidisciplinary integrated design problems.

Acknowledgments

This study was supported by the Flight Vehicle Research Center.

References

[1] HexWeb honeycomb attributes and properties. A comprehensive guide to standard Hexcel honeycomb materials, configurations, and mechanical properties. HEXCEL Composites. 1999.

[2] Long MW, Narciso JD. Probabilistic design methodology for composite aircraft structures. Washington, (DC): U.S. Department of Transportation/Federal Aviation Administration. 1999; DOT/FAA/AR-99/2.

[3] Haldar A, Mahadevan S. Probability, reliability, and statistical methods in engineering design. New York (NY): Wiley; 2000.
[4] Haldar A, Mahadevan S. Reliability assessment using stochastic finite element analysis. New York (NY): Wiley; 2000.
[5] Niu MCY. Airframe; stress analysis and sizing. 2nd ed. Hong Kong: Conmilit Press; 2000. p. 24–74.
[6] Corke Thomas C. Design of aircraft. Upper Saddle River (NJ): Pearson Education; 2003. p. 214–215.
[7] Choi KK, Youn BD. On probabilistic approaches for reliability-based design optimization (RBDO). In: 9th AIAA/ISSMO Symposium on Multidisciplinary Analysis and Optimization; 2002 Sep 4–6; Atlanta, GA. AIAA-2002–5472.
[8] Agarwal H, Renaud JE. Reliability based design optimization for multidisciplinary systems using response surfaces. In: 43rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference; 2002 Apr 22–25; Denver, CO. AIAA-2002-1755.
[9] Bichon BJ, Mahadevan S, Eldred MS. Reliability-based design optimization using efficient global reliability analysis. In: 50th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference; 2009 May 4–7; Palm Springs, CA. AIAA-2009-2261.
[10] Lee SJ, Kim IG. A study on modified reliability-based optimization method for composite structures. In: Proceedings of the 8th Korea-Japan Joint Symposium on Composite Materials; Changwon, Korea; 2011.
[11] Yuan C, Bergsma O, Koussios S, Zu L, Beukers A. Optimization of sandwich composites fuselages under flight loads. Appl. Compos. Mater. 2010;19.1:47–64.
[12] Jeong HK, Shenoi RA. Probabilistic strength analysis of rectangular FRP plates using Monte Carlo simulation. Comput. Struct. 2000;76:219–235.
[13] Petras A. Design of sandwich structures [Doctoral dissertation]. Cambridge: Robinson College, Cambridge University; 1998.